

Coupled Simulation of Deformable Bodies and ISPH Fluids for Secondary Bone Healing

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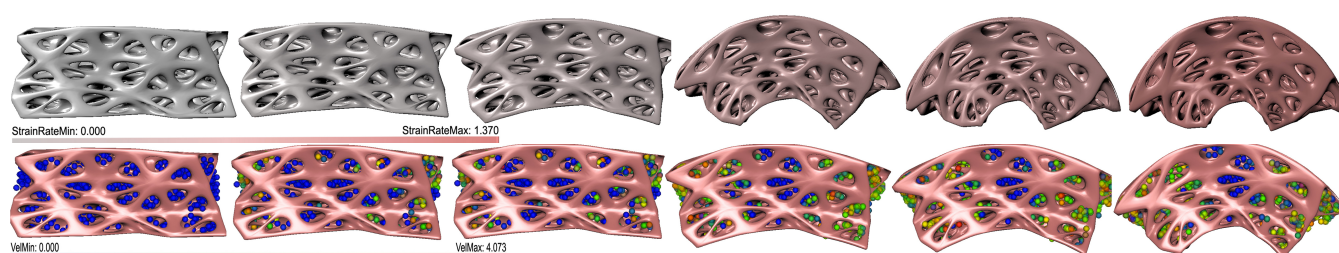


Figure 1: Top row: deformation rate; an animated sequence of increasing levels in strain deformation ranging from 0.000 to 1.370. Bottom row: flux velocity; an animated sequence that illustrates a fluid particles' velocity during the soft tissues deformation, and shows the diffusion of the fluids within the tissues. In the secondary bone healing process, this flux allows nutrient transportation through cartilages, which influences the healing process significantly.

Abstract

We propose a new scheme for the two-way coupling of incompressible fluids and deformable bodies, where we focus on a medical application; in particular, secondary bone healing. Our method allows for accurate simulation and visualisation of the secondary bone healing process, which is used to optimise clinical treatment of bone fractures. In our simulation, the soft tissues are simulated as elastic materials using Strain Based Dynamics (SBD), and fluid is simulated using Incompressible Smoothed Particle Hydrodynamics (ISPH). The interaction model we propose works with any type of deformation technique as long as the object surface is represented by a polygonal mesh and the fluid by Lagrangian particles.

Categories and Subject Descriptors (according to ACM CCS): I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism—Animation and Virtual Reality

1. Introduction

Simulation of the physical interaction between fluid and deformable bodies is of great importance to numerous applications in science, engineering, medicine, and entertainment. However, producing simultaneously the elastic behaviour of the deformable body and fluid phenomena can be complex and computationally demanding. In the present work, we study this problem for a medical application. In particular, we present a new simple yet stable method for simulating the interaction between elastic material and fluid in secondary bone healing process. The goal of the simulation itself is to investigate the mechanical stimuli driving tissue development during secondary bone healing to optimise clinical treatment of bone fractures. The materials used by the body to repair the fracture via secondary healing are bone, cartilage and soft tissue. The cellular

structure of the soft tissues is made of an interconnected network of solid struts or plates, which form the cell's edges and faces. To adapt the tissue's material properties, it is simulated as non-linear elastic materials using Strain Based Dynamics (SBD) [MCKM14]. The cellular materials of the tissues are modelled as permeable to fluid flow, while fluid is allowed to occur across borders of the model to imitate the continuity of the callus (Fig. 1). Mechanical stimuli produce relatively small pore pressures on the tissue, which cause shear deformation, as well as volumetric deformation, which is expected to increase fluid flows in the fracture site. Fluid is simulated using Incompressible Smoothed Particle Hydrodynamics (ISPH). SBD is based on the Position Based Dynamics framework (PBD) [MHHR07], which works in real-time and is widely used to provide appealing deformations [ARF15]. The use of SBD to

model the elasticity of deformable bodies allows us to work with the strain directly by deriving positional constraints for each entry of the strain tensor. This is unlike e.g. Finite Element Methods (FEMs), which first derive stresses from strains, then integrate the deformation energy and finally derive forces. SBD offers stability and room for extension to simulate interesting interaction phenomena between fluids and rigid bodies. The resulting algorithm is general and has a relatively simple formulation inside the PBD schema.

Smoothed Particle Hydrodynamics (SPH) [Mon92] is a Lagrangian mesh-free model that allows us to solve the Navier-Stokes equations for a set of interacting fluid particles. While the original SPH method achieves incompressibility by a weak compressibility approach, ISPH simulates incompressible flows preserving the volume [ESE07, RSB15].

2. Coupled Simulation of Deformable Bodies and ISPH Fluids

Introducing a local mechanical stimulus within the developing calculus tissue can influence cellular differentiation, and fluid diffusion. The latter occurs according to the soft tissue's volume deformations. During the simulation, the fluid particles move from an area of high to low concentration (Fig. 2). This fluid flux allows nutrient transportation through cartilages, which influences the healing process significantly.

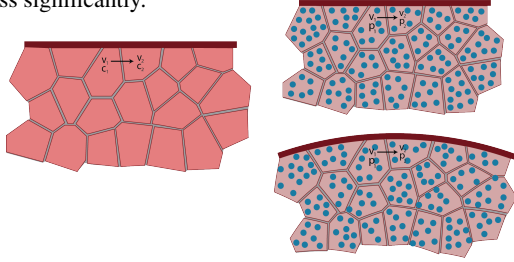


Figure 2: Soft tissue deformations lead to change in volume v and produce pressure p , which forces fluid particles to travel from an area of high to low concentration c .

2.1. Strain Based Dynamics

Strain Based Dynamics derives a set of positional projections for the deformation modes corresponding to the entries of the Green-St Venant strain tensor. SBD introduces new constraints within the PBD schema that involve all three particles in a triangle mesh to be driven into a state in which the components of Green's strain tensor assume given values. In SBD, the formulation of these constraints uses strain components in terms of the positions of the three particles adjacent to the triangle, where the deformation gradient $\mathbf{F}$ and Green-St Venant strain tensor $\mathbf{G}$ are defined as the following:

$$\mathbf{F} = \mathbf{PQ}^{-1}, \quad \mathbf{G} = \mathbf{F}^T \mathbf{F} - \mathbf{I} \quad (1)$$

where Q is material positions, P is the corresponding world positions and I is the identity matrix.

2.2. Incompressible Smoothed Particle Hydrodynamics (ISPH)

SPH is a particle-based method, where Navier-Stokes equations govern the conservation of momentum, while the continuity equation states the conservation of mass.

$$\frac{d\mathbf{u}}{dt} = \frac{1}{\rho} (-\nabla p + \nabla \cdot (2\mu \mathbf{D})) + \mathbf{f}^B \quad (2)$$

where $\mathbf{u}$ is the velocity, p is the pressure, ρ and μ are the density and coefficient of viscosity of the fluid, respectively. $\mathbf{D} = (\nabla \mathbf{u} + \nabla \mathbf{u}^T)/2$ is the deformation rate tensor, $\mathbf{f}^B$ is the body force per unit mass on the fluid element and t is the time. For incompressible flows, the material derivative of density is zero, and therefore mass conservation equation is given by,

$$\nabla \cdot \mathbf{u} = 0 \quad (3)$$

2.3. Two-way Coupling of Fluids and Deformable bodies

The behaviour of fluids is limited by boundary conditions that relate the quantities of the two adjacent materials at the interface. In the case of fluid-deformable bodies interaction, the geometrical domain of the interface is defined as a surface between the deformable mesh continuum and the volumetric fluid continuum. Therefore, the distance between a fluid particle and a triangle or a particle and a triangulated surface needs to be defined. Let us define the distance of a particle $\mathbf{p}$ from a triangle t as:

$$d(\mathbf{p}, t) = \min |\mathbf{p} - \mathbf{x}|, \quad \text{where } \mathbf{x} \in t \quad (4)$$

and the distance of a point p from a triangulated surface T :

$$d(\mathbf{p}, T) = \min d(\mathbf{p}, t), \quad \text{where } t \in T \quad (5)$$

For every interaction pair, composed of a boundary triangle and a fluid particle, forces are computed and applied to related triangles and fluid particles. We use a Lennard-Jones-like force to generate repulsive forces which approximate the no-penetration condition. The force acting on particle $\mathbf{p}$ due to triangle t is defined as follows:

$$\mathbf{f}(\mathbf{p}, t) = \int_{x \in t} \tau(|\mathbf{p} - \mathbf{x}|) d\mathbf{x} \quad (6)$$

The traction τ is dependent on the distance of the surface element from the particle $\mathbf{p}$ and has unit force per unit of area in order to yield a force when integrated over the triangle.

3. Discussion and Conclusion

We propose a practical schema to simulate the interactions between particle-based fluids and mesh-based deformable bodies for secondary bone healing. Our method is simple, easy to implement and can be useful in many graphic domains. It enables the use of distinct simulation methods for the soft tissue and the fluid. In particular, it does not impose the use of only particle methods for both the soft tissue and the fluid.

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